CHAPTER 8

Mechanical Properties of Solids and Fluids

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. A wire of length L and cross sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is .... *(1987 - 2 Marks)*

2. A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless pistion of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid the fractional change in the radius of the sphere, $\delta R/R$, is .... *(1988 - 2 Mark)*

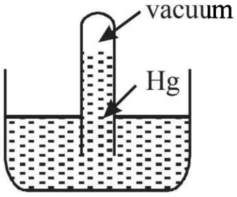
3. A piece of metal floats on mercury. The coefficients of volume expansion of the metal and mercury are γ_1 and γ_2 respectively. If the temperatures of both mercury and the metal are increased by an amount ΔT, the fraction of the volume of the metal submerged in mercury changes by the factor .... *(1991 - 2 Mark)*

4. A horizontal pipeline carries water in a streamline flow. At a point along the pipe, where the cross- sectional area is 10 cm^2, the water velocity is 1 ms^{-1} and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2, is... Pa. (Density of water = 10^3 $kg.m^{-3}$) *(1994 - 2 Marks)*

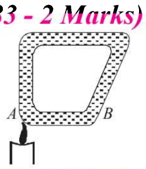
B True/False

1. A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, the level of the water in the pond decreases. *(1980)*

2. A barometer made of a very narrow tube (see Fig) is placed at normal temperature and pressure. The coefficient of volume expansion of mercury is 0.00018 per C° and that of the tube is negligible. The temperature of mercury in the barometer is now raised by 1°C, but the temperature of the atmosphere does not change. Then the mercury height in the tube remains unchanged. *(1983 - 2 Marks)*



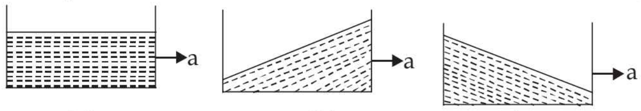
3. Water in a closed tube (see Fig) is heated with one arm vertically placed above a lamp. Water will begin to circulate along the tube in counter-clockwise direction. *(1983 - 2 Marks)*



4. A block of ice with a lead shot embedded in it is floating on water contained in a vessel. The temperature of the system is maintained at 0°C as the ice melts. When the ice melts completely the level of water in the vessel rises . *(1986 - 3 Marks)*

C MCQs with One Correct Answer

1. A vessel containing water is given a constant acceleration 'a' towards the right along a straight horizontal path. Which of the following diagrams in Fig. represents the surface of the liquid ? *(1981- 2 Marks)*

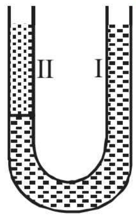


(a) (b) (c)

2. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ? *(1981- 2 Marks)*

(a) length = 50 cm , diameter = 0.5 mm
(b) length = 100 cm, diameter = 1 mm
(c) length = 200 cm, diameter = 2 mm
(d) length = 300 cm, diameter = 3 mm.

3. A U-tube of uniform cross section (see Fig) is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be *(1983 - 1 Mark)*



(a) 1.12
(b) 1.1
(c) 1.05
(d) 1.0

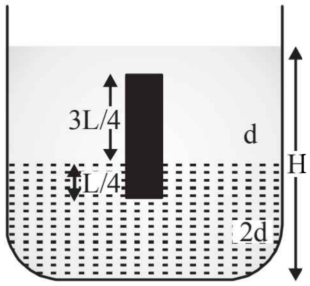
4. A homogeneous solid cylinder of length L ($L < H/2$), cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure P_0. Then density D of solid is given by *(1995S)*

(a) $\frac{5}{4}d$

(b) $\frac{4}{5}d$

(c) $4d$

(d) $\frac{d}{5}$



5. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to *(2000S)*

(a) $\frac{L}{\sqrt{2\pi}}$ (b) $2\pi L$ (c) L (d) $\frac{L}{2\pi}$

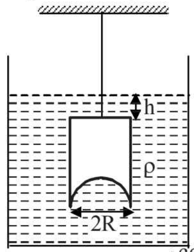
6. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is *(2001S)*

(a) Mg

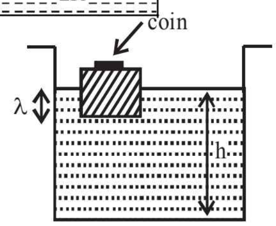
(b) $Mg - V\rho g$

(c) $Mg + \pi R^2 h\rho g$

(d) $\rho g(V + \pi R^2 h)$



7. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance ℓ and h are shown here. After some time the coin falls into the water. Then



(a) ℓ decreases and h increases *(2002S)*
(b) ℓ increases and h decreases
(c) both ℓ and h increase (d) both ℓ and h decrease

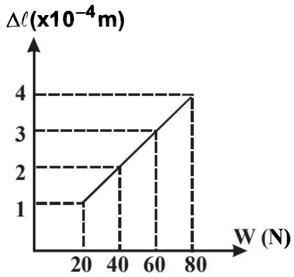
8. The adjacent graph shows the estension ($\Delta\ell$) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire. *(2003S)*

(a) 2×10^{11} N/m

(b) 2×10^{-11} N/m

(c) 3×10^{-12} N/m

(d) 2×10^{-13} N/m



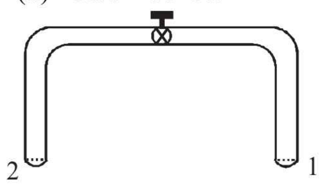
9. Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $a/A = 0.1$ then v^2 is (where v is the velocity of water coming out of the hole) *(2005S)*

(a) 50 (b) 51 (c) 48 (d) 51.5

10. When temperature of a gas is 20°C and pressure is changed from $p_1 = 1.01 \times 10^5$ Pa to $p_2 = 1.165 \times 10^5$ Pa then the volume changed by 10%. The bulk modulus is *(2005S)*

(a) 1.55×10^5 Pa (b) 0.115×10^5 Pa
(c) 1.4×10^5 Pa (d) 1.01×10^5 Pa

11. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position.



End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve, *(2008)*

(a) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
(b) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
(c) no changes occurs
(d) air from end 2 flows towards end 1. volume of the soap bubble at end 1 increases

12. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is *(2012- II)*

(a) more than half-filled if ρ_c is less than 0.5.
(b) more than half-filled if ρ_c is more than 1.0.
(c) half-filled if ρ_c is more than 0.5.
(d) less than half-filled if ρ_c is less than 0.5.

13. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(a) 0.25 (b) 0.50 *(JEE Adv. 2013)*
(c) 2.00 (d) 4.00

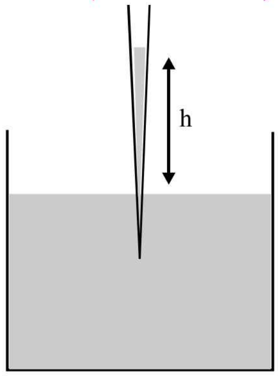
14. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity) *(JEE Adv. 2014)*

(a) $\frac{2S}{b\rho g}\cos(\theta - \alpha)$

(b) $\frac{2S}{b\rho g}\cos(\theta + \alpha)$

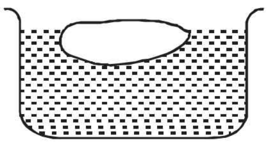
(c) $\frac{2S}{b\rho g}\cos(\theta - \alpha/2)$

(d) $\frac{2S}{b\rho g}\cos(\theta + \alpha/2)$



D MCQs with One or More than One Correct

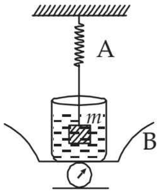
1. A body floats in a liquid contained in a beaker. The whole system as shown in Figure falls freely under gravity. The upthrust on the body is *(1982 - 3 Marks)*



(a) zero
(b) equal to the weight of the liquid displaced
(c) equal to the weight of the body in air
(d) equal to the weight of the immersed portion of the body

2. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation: *(1985 - 2 Marks)*

(a) the balance A will read more than 2 kg
(b) the balance B will read more than 5 kg
(c) the balance A will read less than 2 kg and B will read more than 5 kg
(d) the balance A and B will read 2 kg and 5 kg respectively



3. A vessel contains oil (density = 0.8 gm/cm^3) over mercury (density = 13.6 gm cm^3). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm/cm^3 is *(1988 - 2 Mark)*

(a) 3.3 (b) 6.4
(c) 7.2 (d) 12.8

4. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's modulii Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2$ = 2 : 3, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to *(1989 - 2 Mark)*

(a) 2 : 3 (b) 1 : 1
(c) 3 : 2 (d) 4 : 9

5. Water from a tap emerges vertically downwards with an initial spped of 1.0 m s^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross-sectional area of the stream 0.15 m below the tap is *(1998S - 2 Marks)*

(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-5} m^2
(c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

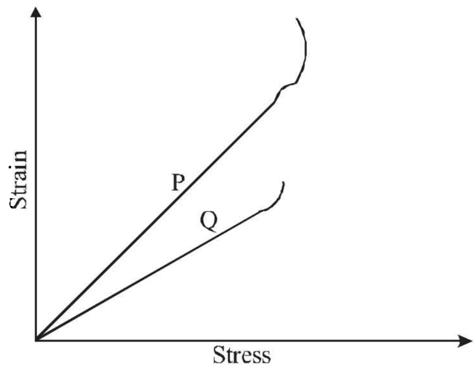
6. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) *(JEE Adv. 2013)*

(a) The net elongation of the spring is $\dfrac{4\pi R^3 \rho g}{3k}$

(b) The net elongation of the spring is $\dfrac{8\pi R^3 \rho g}{3k}$

(c) The light sphere is partially submerged
(d) The light sphere is completely submerged

7. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is (are) *(JEE Adv. 2015)*



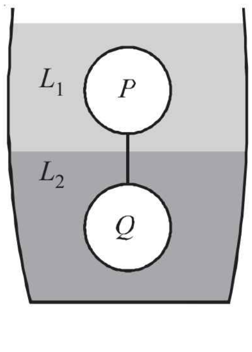
(a) P has more tensile strength than Q
(b) P is more ductile than Q
(c) P is more brittle than Q
(d) The Young's modulus of P is more than that of Q

8. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct option(s) is (are) *(JEE Adv. 2015)*

(a) $P(r=0)=0$ (b) $\dfrac{P(r=3R/4)}{P(r=2R/3)}=\dfrac{63}{80}$

(c) $\dfrac{P(r=3R/5)}{P(r=2R/5)}=\dfrac{16}{21}$ (d) $\dfrac{P(r=R/2)}{P(r=R/3)}=\dfrac{20}{27}$

9. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then *(JEE Adv. 2015)*



(a) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|}=\dfrac{\eta_1}{\eta_2}$

(b) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|}=\dfrac{\eta_2}{\eta_1}$

(c) $\vec{V}_P . \vec{V}_Q > 0$

(d) $\vec{V}_P . \vec{V}_Q < 0$

E Subjective Problems

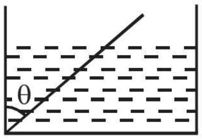
1. A column of mercury of 10 cm length is contained in the middle of a narrow horizontal 1 m long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance will the column of mercury be displaced if the tube is held vertically? *(1978)*

2. A point mass m is suspended at the end of a massless wire of length l and cross section A. If Y is the Young's modulus for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line. *(1978)*

3. A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube ruses by 2cm. What is the size of the cube? *(1978)*

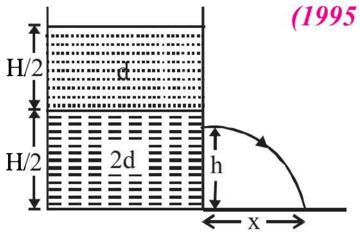
4. A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level? *(1979)*

5. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in fig The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0°) *(1984- 8 Marks)*



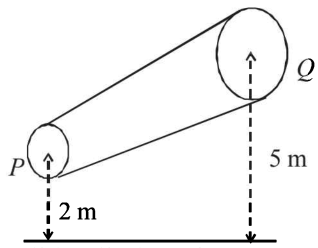
6. A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density of d_L *(1992 - 8 Marks)*
   (a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.
   (b) Is the motion of the ball simple harmonic?
   (c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

7. A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $H/2$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0. *(1995 - 5 + 5 Marks)*

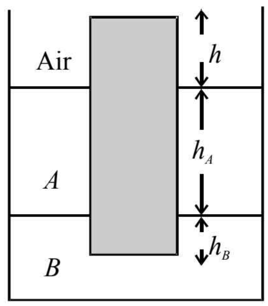


   (a) A homogeneous solid cylinder of length L $(L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid. Determine:
      (i) the density D of the solid and
      (ii) the total pressure at the bottom of the container.
   (b) The cylinder is removed and the original arrangement is restored. A tiny hole of area s $(s << A)$ is punched on the vertical side of the container at a height h $(h < H/2)$. Determine :
      (i) the initial speed of efflux of the liquid at the hole,
      (ii) the horizontal distance x travelled by the liquid initially, and
      (iii) the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m.
      (Neglect the air resistance in these calculations.)

8. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in Figure. The area of cross section of the tube two points P and Q at heights of 2 metres and 5 metres are respectively $4{\times}10^{-3}$ m^2 and $8{\times}10^{-3}$ m^2. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. *(1997 - 5 Marks)*

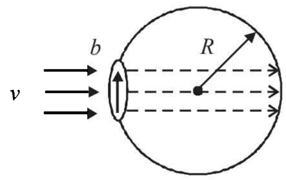


9. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical.
   The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm. *(2002 - 5 Marks)*

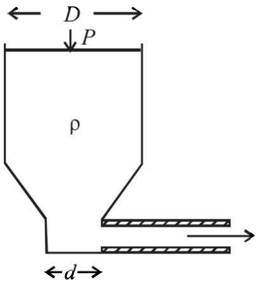


   (a) Find the total force exerted by liquid A on the cylinder.
   (b) Find h, the length of the part of the cylinder in air.
   (c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.

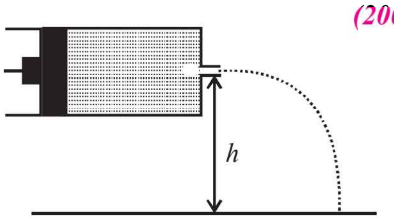
10. A bubble having surface tension T and radius R is formed on a ring of radius b $(b << R)$. Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring. *(2003 - 4 Marks)*



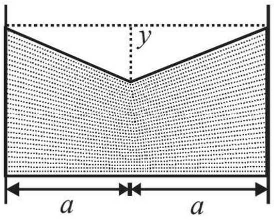
11. Shown in the figure is a container whose top and bottom diameters are D and d respectively. At the bottom of the container, there is a capillary tube of outer radius b and inner radius a. The volume flow rate in the capillary is Q. If the capillary is removed the liquid comes out with a velocity of v_0. The density of the liquid is given as ρ. Calculate the coefficient of viscosity η. *(2003 - 4 Marks)*



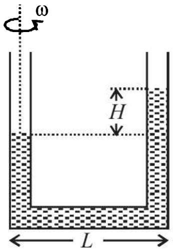
12. A tube has two area of cross-sections as shown in figure. The diameters of the tube are 8 mm and 2 mm. Find range of water falling on horizontal surface, if piston is moving with a constant velocity of 0.25 m/s, $h = 1.25$ m ($g = 10$ m/s^2) *(2004 - 2 Marks)*



13. A uniform wire having mass per unit length λ is placed over a liquid surface. The wire causes the liquid to depress by $y (y << a)$ as shown in figure. Find surface tension of liquid. Neglect end effect. *(2004 - 2 Marks)*



14. A U tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube $d << L$. *(2005 - 2 Marks)*



F Match the Following

DIRECTIONS (Q. No. 1) : *Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. A person in lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List-I and the distance where the water jet hits the floor of the lift is given in List-II. Match the statements from List-I with those in List-II and select the correct answer using the code given below the lists. *(JEE Adv. 2014)*

	List - I		List - II
P.	Lift is accelerating vertically up	**1.**	$d = 1.2$ m
Q.	Lift is accelerating vertically down with an acceleration less than the gravitational acceleration	**2.**	$d > 1.2$ m
R.	Lift is moving vertically up with constant speed	**3.**	$d < 1.2$ m
S.	Lift is falling freely	**4.**	No water leaks out of the jar

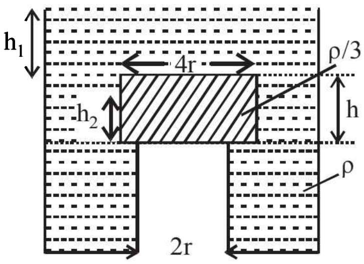
Code:

(a) P-2, Q-3, R-2, S-4 (b) P-2, Q-3, R-1, S-4 (c) P-1, Q-1, R-1, S-4 (d) P-2, Q-3, R-1, S-1

G Comprehension Based Questions

PASSAGE-I

A cylindrical tank has a hole of diameter $2r$ in its bottom. The hole is covered wooden cylindrical block of diameter $4r$, height h and density $\rho/3$.



Situation I : Initially, the tank is filled with water of density ρ to a height such that the height of water above the top of the block is h_1 (measured from the top of the block).

Situation II : The water is removed from the tank to a height h_2 (measured from the bottom of the block), as shown in the figure. The height h_2 is smaller than h (height of the block) and thus the block is exposed to the atmosphere.

1. Find the minimum value of height h_1 (in situation 1), for which the block just starts to move up? *(2006 - 5M, –2)*

(a) $\frac{2h}{3}$ (b) $\frac{5h}{4}$ (c) $\frac{5h}{3}$ (d) $\frac{5h}{2}$

2. Find the height of the water level h_2 (in situation 2), for which the block remains in its original position without the application of any external force *(2006 - 5M, –2)*

(a) $\frac{h}{3}$ (b) $\frac{4h}{9}$ (c) $\frac{2h}{3}$ (d) h

3. In situation 2, if h_2 is further decreased, then *(2006 - 5M, –2)*

(a) cylinder will not move up and remains at its original position

(b) for $h_2 = \frac{h}{3}$, cylinder again starts moving up

(c) for $h_2 = \frac{h}{4}$, cylinder again starts moving up

(d) for $h_2 = \frac{h}{5}$, cylinder again starts moving up

PASSAGE-II

When liquid medicine of density ρ is to put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

4. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is *(2010)*

(a) $2\pi rT$ (b) $2\pi RT$

(c) $\frac{2\pi r^2 T}{R}$ (d) $\frac{2\pi R^2 T}{r}$

5. If $r = 5 \times 10^{-4}\,\text{m}, \rho = 10^3\,\text{kgm}^{-3}, g = 10\,\text{ms}^{-2}, T = 0.11\,\text{Nm}^{-1}$, the radius of the drop when it detaches from the dropper is approximately *(2010)*

(a) $1.4 \times 10^{-3}\,\text{m}$ (b) $3.3 \times 10^{-3}\,\text{m}$

(c) $2.0 \times 10^{-3}\,\text{m}$ (d) $4.1 \times 10^{-3}\,\text{m}$

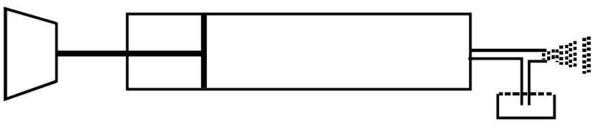
6. After the drop detaches, its surface energy is *(2010)*

(a) $1.4 \times 10^{-6}\,\text{J}$ (b) $2.7 \times 10^{-6}\,\text{J}$

(c) $5.4 \times 10^{-6}\,\text{J}$ (d) $8.1 \times 10^{-6}\,\text{J}$

PASSAGE-III

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



7. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of *(JEE Adv. 2014)*

(a) 0.1 ms^{-1} (b) 1 ms^{-1}

(c) 2 ms^{-1} (d) 8 ms^{-1}

8. If the density of air is ρ_a, and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to *(JEE Adv. 2014)*

(a) $\sqrt{\frac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$ (c) $\sqrt{\frac{\rho_l}{\rho_a}}$ (d) ρ_l

H Assertion & Reason Type Questions

1. **STATEMENT-1 :** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

STATEMENT-2 : In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. *(2008)*

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

I Integer Value Correct Type

1. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.] *(2009)*

2. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.
[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.] *(2009)*

3. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its crosssectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is n × 10^9 Nm^{-2}, the value of n is *(2010)*

4. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is *(JEE Adv. 2016)*

Section-B JEE Main / AIEEE

1. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(a) 16 J (b) 8 J [2002]
(c) 32 J (d) 24 J

2. A wire fixed at the upper end stretches by length ℓ by applying a force F. The work done in stretching is [2004]
(a) $2F\ell$ (b) $F\ell$ (c) $\frac{F}{2\ell}$ (d) $\frac{F\ell}{2}$

3. Spherical balls of radius 'R' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is [2004]
(a) inversely proportional to both radius 'R' and velocity 'v'
(b) directly proportional to both radius 'R' and velocity 'v'
(c) directly proportional to 'R' but inversely proportional to 'v'
(d) inversely proportional to 'R' but directly proportional to velocity 'v'

4. If two soap bubbles of different radii are connected by a tube. [2004]
(a) air flows from the smaller bubble to the bigger
(b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(d) there is no flow of air.

5. If 'S' is stress and 'Y' is young's modulus of material of a wire, the energy stored in the wire per unit volume is
(a) $\frac{S^2}{2Y}$ (b) $2S^2Y$ [2005]
(c) $\frac{S}{2Y}$ (d) $\frac{2Y}{S^2}$

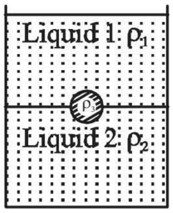
6. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be [2005]
(a) 10 cm (b) 8 cm (c) 20 cm (d) 4 cm

7. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm) [2006]
(a) l (b) $2l$ (c) zero (d) $l/2$

8. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid [2006]
(a) 0.4 m/s (b) 0.133 m/s (c) 0.1 m/s (d) 0.2 m/s

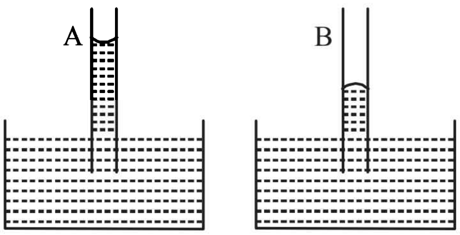
9. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is
(a) $\sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$ (b) $\frac{Vg\rho_1}{k}$ [2008]
(c) $\sqrt{\frac{Vg\rho_1}{k}}$ (d) $\frac{Vg(\rho_1-\rho_2)}{k}$

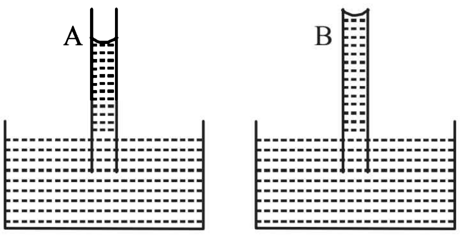
10. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and, ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_1 and ρ_3? [2008]

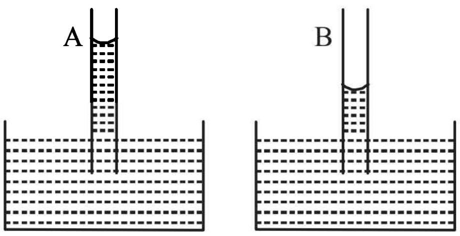


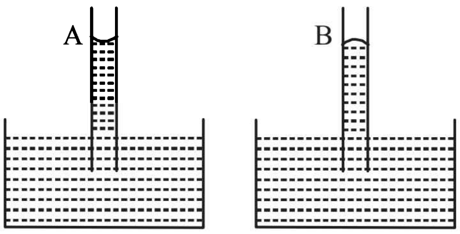
(a) $\rho_3 < \rho_1 < \rho_2$
(b) $\rho_1 > \rho_3 > \rho_2$
(c) $\rho_1 < \rho_2 < \rho_3$
(d) $\rho_1 < \rho_3 < \rho_2$

11. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? [2008]

(a)


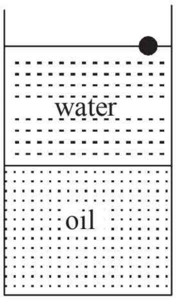
(b)


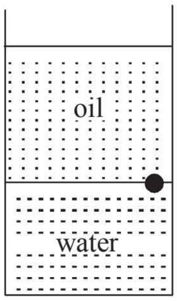
(c)


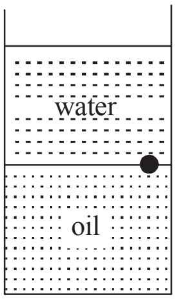
(d)


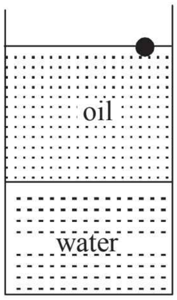
12. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? [2009]
(a) $4F$ (b) $6F$ (c) $9F$ (d) F

13. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ? [2010]

(a)


(b)


(c)


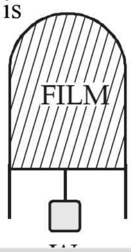
(d)


14. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is [2010]
(a) 4 (b) 3
(c) 2 (d) 1

15. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) [2011]
(a) 0.2πmJ (b) 2πmJ
(c) 0.4πmJ (d) 4πmJ

16. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to: [2011]
(a) 7.5×10^{-3} m (b) 9.6×10^{-3} m
(c) 3.6×10^{-3} m (d) 5.0×10^{-3} m

17. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is [2012]
(a) 0.0125 Nm^{-1}
(b) 0.1 Nm^{-1}
(c) 0.05 Nm^{-1}
(d) 0.025 Nm^{-1}



18. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is: [JEE Main 2013]

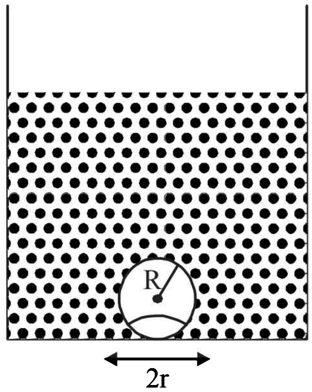
(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

(c) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

19. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization. [JEE Main 2013]

(a) $\rho L/T$ (b) $\sqrt{T/\rho L}$
(c) $T/\rho L$ (d) $2T/\rho L$

20. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r ≪ R and the surface tension of water is T, value of r just before bubbles detach is:
(density of water is ρ_w) [JEE Main 2014]



(a) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

21. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) [JEE Main 2014]
(a) 16 cm (b) 22 cm
(c) 38 cm (d) 6 cm

8 Mechanical Properties of Solids and Fluids

Section-A : JEE Advanced/ IIT-JEE

A 1. $\frac{YAx^2}{2L}$ 2. $\frac{Mg}{3Ak}$ 3. $(\gamma_2-\gamma_1)\Delta T$ 4. 500 Pa.

B 1. F 2. F 3. F 4. F

C 1. (c) 2. (a) 3. (b) 4. (a) 5. (a) 6. (d)
7. (d) 8. (a) 9. (a) 10. (a) 11. (b) 12. (a)
13. (c) 14. (d)

D 1. (a) 2. (b, c) 3. (c) 4. (c) 5. (c) 6. (a, d)
7. (b, c) 8. (a, b) 9. (a, d)

E 1. 2.95 cm 2. $\frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ 3. $l = 10$ cm 4. Fall 5. 45° 6. **(a)** $\frac{d_L t_1}{d_L - d}$
(b) no **(c)** remains same

7. **(a) (i)** $\frac{5d}{4}$ **(ii)** $P_0+\left(\frac{3H}{2}+\frac{L}{4}\right)dg$ **(b) (i)** $\frac{\sqrt{3H-4h}}{2}g$ **(ii)** $\sqrt{(3H-4h)h}$ **(iii)** $\frac{3H}{8}, \frac{3H}{4}$ 8. 29.025×10^3 J/m^3 ; 29.4×10^3 J/m^3 9. **(a)** zero **(b)** 0.25 cm **(c)** $g/6,\uparrow$ 10. $\frac{4T}{\rho v^2}$ 11. $\frac{\pi}{8Q\ell}\times\frac{1}{2}\rho v_0^2\left[1-\frac{d^4}{D^4}\right]\times a^4$

12. 2m 13. $\frac{\lambda a g}{2y}$ 14. $H=\frac{\omega^2 L^2}{2g}$

F 1. (c)

G 1. (c) 2. (b) 3. (a) 4. (c) 5. (a) 6. (b)
7. (c) 8. (a)

H 1. (a)

I 1. 6 2. 6 3. 4 4. 3

Section-B : JEE Main/ AIEEE

1. (b) 2. (d) 3. (b) 4. (a) 5. (a) 6. (c) 7. (a) 8. (c)
9. (a) 10. (d) 11. (c) 12. (c) 13. (b) 14. (c) 15. (c) 16. (c)
17. (d) 18. (c) 19. (d) 20. (None) 21. (a)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. $W=\frac{1}{2}\times Y\times(\text{strain})^2\times Yd=\frac{1}{2}\times Y\times\frac{x^2}{L^2}\times AL=\frac{YAx^2}{2L}$

2. $K=\frac{-\Delta P}{\Delta V/V}$

where $\Delta P=\frac{Mg}{A}$ $\therefore$ $-\frac{\Delta V}{V}=\frac{Mg}{AK}$

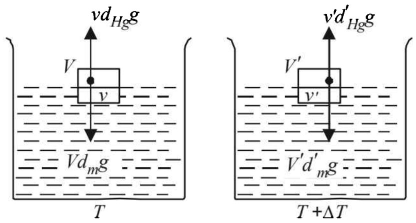
$\Rightarrow -\frac{(V_f-V_i)}{V_i}=\frac{Mg}{AK}$ $\Rightarrow \frac{V_i-V_f}{V_i}=\frac{Mg}{AK}$

$$\Rightarrow \frac{\frac{4}{3}\pi R^3-\frac{4}{3}\pi(R-\delta R)^3}{\frac{4}{3}\pi R^3}=\frac{Mg}{AK}$$

$$\Rightarrow \frac{R^3-[R^3-3R^2\delta R]}{R^3}=\frac{Mg}{AK} \Rightarrow \frac{\delta R}{R}=\frac{Mg}{3AK}$$

3. **KEY CONCEPT**
Using the relation for floatation, $vd_{Hg}g = Vd_m g$
Fraction of volume of metal submerged in mercury

$$=\frac{v}{V}=\frac{d_m}{d_{Hg}}=K_1 \text{ (say)}$$



In second case, when temperature is increased by ΔT.

$$v'd'_{Hg}g = V'd'_m g$$

$$\Rightarrow \frac{v'}{V'} = \frac{d'_m}{d'_{Hg}} = \text{Fraction of volume of metal submerged}$$

in mercury $= K_2$ (say)

$$\therefore \frac{K_2}{K_1} = \frac{d'_m \times d_{Hg}}{d'_{Hg} \times d_m} = \frac{d'_m \times d'_{Hg}(1+\gamma_2 \Delta T)}{d'_{Hg} \times d'_m (1+\gamma_1 \Delta T)} = \frac{(1+\gamma_2 \Delta T)}{(1+\gamma_1 \Delta T)}$$

$$= (1+\gamma_2 \Delta T)(1+\gamma_1 \Delta T)^{-1}$$

$$= (1+\gamma_2 \Delta T)(1-\gamma_1 \Delta T) = 1 + (\gamma_2 - \gamma_1)\Delta T$$

Note : If $\gamma_2 - \gamma_1$ then $k_2 > k_1$

i.e., metal block will get immersed deeper

If $\gamma_2 < \gamma_1$ then $k_2 < k_1$

i.e. metal block will rise a bit as compared to its previous position.

$$\frac{K_2}{K_1} - 1 = (\gamma_2 - \gamma_1)\Delta T \Rightarrow \frac{K_2 - K_1}{K_1} = (\gamma_2 - \gamma_1)\Delta T$$

4. **KEY CONCEPT**

Applying equation of continuity at cross section 1 and 2

$$A_1 v_1 = A_2 v_2 \Rightarrow 10 \times 1 = 5 \times v_2 \Rightarrow v_2 = 2\text{m/s}$$

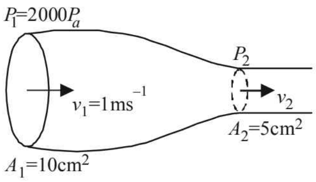
Applying Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow 2000 + \frac{1}{2} \times 1000 \times 1^2$$

$$= P_2 + \frac{1}{2} \times 1000 \times 2^2$$

$$\Rightarrow P_2 = 500\ Pa$$



B. True/ False

1. When the man drinks some water from the pond, his weight increases and therefore the boat will sink further. The further sinking of the boat will displace the same volume of water in pond as drunk by man. Therefore, there will no change in the level of water in the pond.

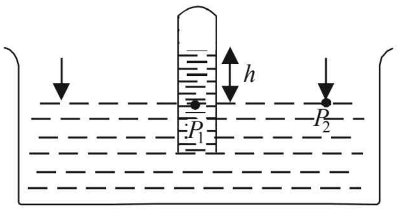
2. Pressure $P_1 = P_2 = 1$ atm $= h\rho g$

Note :

On changing the temperature, g will not change and atmospheric pressure will not change.

$\therefore$ $h \times \rho =$ constant.

When temperature is increased, the density of Hg decreases and hence, h increases.

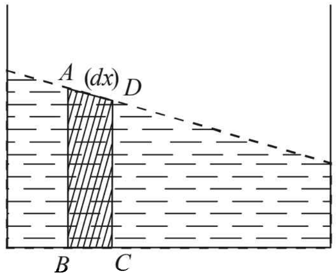


3. When water is heated at end A, the density decreases and the water moves up. This is compensated by the movement of water from B to A i.e., in clockwise direction.

4. When the block of ice melts, the lead shot will ultimately sink in the water. When lead shot sinks, it will displace water equal to its own volume. But when lead shot was embedded in ice, it displaced more volume of water than its own volume because $d_{lead} > d_{water}$. Therefore, level of water will fall.

C. MCQs with ONE Correct Answer

1. **(c)**



Let us consider a small dotted segment of thickness dx for observation.

Since, this segment is accelerated towards right, a net force is acting in this segment towards right from the liquid towards the left of $ABCD$. According to Newton's third law, the segment $ABCD$ will also apply a force on the previous section creating a pressure on it which makes the liquid rise.

2. **(a)** $Y = \frac{T/A}{\Delta \ell/\ell} \Rightarrow \Delta \ell = \frac{T \times \ell}{A \times Y} = \frac{T}{Y} \times \frac{\ell}{A}$

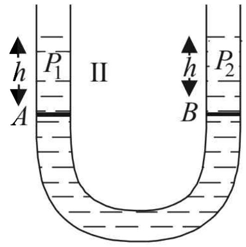
Here, $\frac{T}{Y}$ is constant. Therefore, $\Delta \ell \propto \frac{\ell}{A}$.

$\frac{\ell}{A}$ is largest in the first case.

3. **(b)** Pressure in limb I at B = Pressure in limb II at A

$$h\rho_1 g = h\rho_2 g$$

$$\Rightarrow \rho_1 = \rho_2$$



4. **(a)** Weight of cylinder = Upthrust due to upper liquid + Upthrust due to lower liquid.

$$D\left(\frac{A}{5} \times L \times g\right) = d\left(\frac{A}{5}\right)\left(\frac{3}{4}L\right) g + 2d\left(\frac{A}{5}\right)\left(\frac{L}{4}\right) \times g$$

$$\therefore \quad D = \frac{5d}{4}$$

5. **(a)** Equating the rate of flow, we have

$$\sqrt{(2gy)} \times L^2 = \sqrt{(2g \times 4y)}\, \pi R^2$$

[Flow = (area) × (velocity), velocity = $\sqrt{2gx}$]

where x = height from top

$$\Rightarrow L^2 = 2\pi R^2 \Rightarrow R = \frac{L}{\sqrt{2\pi}}$$

6. **(d)** **KEY CONCEPT :**

According to Archimedes principle

Upthrust = Wt. of fluid displaced

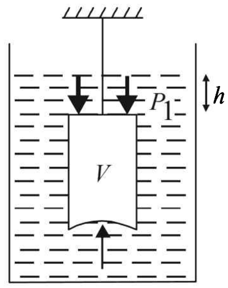
$F_{bottom} - F_{top} = V\rho g$

$\therefore$ $F_{bottom} = F_{top} + V\rho g$

$= P_1 \times A + V\rho g$

$= (h\rho g) \times (\pi R^2) + V\rho g$

$= \rho g\,[\pi R^2 h + V]$



7. **(d)** ℓ decreases as the block moves up. *h* will also decreases because when the coin is in water it will displace a volume of water, equal to its own volume, whereas when it is on the block it displaces more volume than to own volume (because density of coin is greater than density of water).

8. **(a)** $Y = \frac{F}{A} \Big/ \frac{\Delta \ell}{\ell} = \frac{F}{A} \cdot \frac{\ell}{\Delta \ell} = \frac{20 \times 1}{10^{-6} \times 10^{-4}} = 2 \times 10^{11} \text{N/m}^2.$

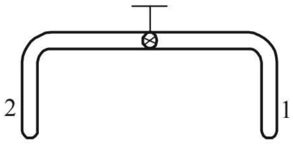
9. **(a)** The square of the velocity of efflux

$$v^2 = \frac{2gh}{\sqrt{1 - \left(\frac{a}{A}\right)^2}} \quad \text{or,} \quad v^2 = \frac{2 \times 10 \times 2.475}{\sqrt{1 - (0.1)^2}} = 50 \text{ m}^2/\text{s}^2$$

$$h = 3 - 0.525 = 2.475 \text{ m}$$

10. **(a)** $B = \frac{\Delta p}{\Delta V / V} = \frac{(1.165 \times 10^5 - 1.01 \times 10^5)}{0.1} = 1.55 \times 10^5 \text{ Pa}$

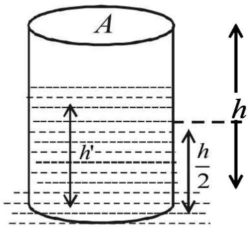
11. **(b)** We know that excess pressure in a soap bubble is inversely proportional to its radius. The soap bubble at end 1 has small radius as compared to the soap bubble at end 2 (given). Therefore excess pressure at 1 is more.



As the value is opened, air flows from end 1 to end 2 and the volume of soap bubble at end 1 decreases.

12. **(a)** Let *V* be the volume of the material of which the cylinder is made. The cylinder is half immersed in water. Therefore the volume of water displaced because of the material of the cylinder is $\frac{V}{2}$. Let *h* be the total height of the cylinder. As the cylinder is half submerged therefore buoyant force

$$B = \frac{V\rho_\omega g}{2} + \frac{hA\rho_\omega g}{2}$$



where *A* is the area of cross-section of the cylinder

The weight of the cylinder $W = V\rho_c g$

The weight of the water inside the cylinder

$= h'A\rho_\omega g$

For equilibrium,

$$\frac{V\rho_\omega g}{2} + \frac{Ah\rho_\omega g}{2} = V\rho_c g + h'A\rho_\omega g$$

Here $\rho_\omega = 1$

$$\therefore \quad h' = \frac{h}{2} + \frac{V}{2A}[1 - 2\rho_c]$$

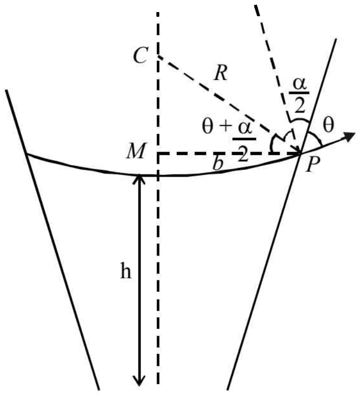
If $\quad \rho_c < 0.5$ then $h' > \frac{h}{2}$

and if $\quad \rho_c > 0.5$ then $h' < \frac{h}{2}$

if $\quad \rho_c = 0, \; h' = \frac{h}{2}$

13. **(c)** $Y = \frac{F/\pi(2R)^2}{\Delta\ell_1 / 2L} = \frac{F/\pi R^2}{\Delta\ell_2 / L} \qquad \therefore \frac{\Delta\ell_2}{\Delta\ell_1} = 2$

14. **(d)**



In ΔPCM $\cos\left(\theta + \frac{\alpha}{2}\right) = \frac{b}{R}$ ...(1)

Also $\left(P_0 - \frac{2S}{R}\right) + h\rho g = P_0$

$$h = \frac{2S}{R\rho g} = \frac{2S}{b\rho g}\cos(\theta + \alpha/2)$$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(a)** The whole system falls freely under gravity

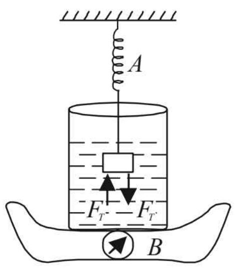
Upthrust = weight of fluid displaced

= (mass of fluid displaced) × *g*

For a freely falling body, $g = 0$

∴ Upthrust = 0.

2. **(b,c)** When the block of mass *m* is arranged as shown in the figure, an upthrust F_T will act on the mass which will decrease the reading on *A*.



Note :

According to Newton's third law, to each and every action, there is equal and opposite reaction.

So F_T will act on the liquid of the beaker which will increase the reading in *B*.

3. **(c)** Weight of sphere

= Upthrust due to Hg + Upthrust due to oil

$$Vdg = \frac{V}{2}d_{Hg}g + \frac{V}{2}d_{oil} \times g$$

$$\Rightarrow \quad d = \frac{d_{Hg} + d_{oil}}{2} = \frac{13.6 + 0.8}{2} = 7.2 \text{g/cm}^3$$

4. **(c)** $\Delta\ell = \ell\alpha\Delta T \Rightarrow \frac{\Delta\ell}{\ell} = \alpha\Delta T$

Stress = Y × strain $\quad \therefore \quad$ Stress = $Y\alpha\Delta T$

For first rod stress = $Y_1\alpha_1\Delta T$

For second rod stress = $Y_2\alpha_2\Delta T$

Since, stresses are equal

$\therefore \quad Y_1\alpha_1\Delta T = Y_2\alpha_2\Delta T$

or, $\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$

5. **(c)** **KEY CONCEPT :**

The equation of continuity is : $v_1 A_1 = v_2 A_2$

where v and A represent the speed of water stream and its area of cross section, respectively. We are given that

$v_1 = 1.0\ ms^{-1}$

$A_1 = 10^{-4}\ m^2$

v_2 = velocity of water stream at 0.15 m below the tap

$A_2 = ?$

For calculating v_2

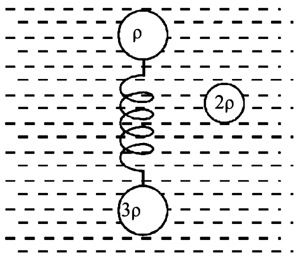
$u = 1\ m/s;\ s = 1.5\ m,\ a = 10\ m/s^2$ and $v = ?$

$v^2 - u^2 = 2as$

$v^2 - 1 = 2 \times 10 \times 0.15 \Rightarrow v = 2\ m/s$

Hence, $A_2 = \dfrac{v_1 A_1}{v_2} = \dfrac{1 \times 10^{-4}}{2} = 5 \times 10^{-5}\ m^2$

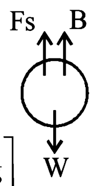
6. **(a, d)** Consider the equilibrium of the system of both spheres and the spring.



The weight of system

$= \dfrac{4}{3}\pi R^3 (3\rho) g + \dfrac{4}{3}\pi R^3 \rho g = 4\left[\dfrac{4}{3}\pi R^3 \rho g\right]$

This is to be balanced by the buoyant force. This can be possible only when the light sphere is completely submerged. In this way the buoyant force



$B = \left[\left(\dfrac{4}{3}\pi R^3\right) \times 2\right] \times (2\rho) \times g = 4\left[\dfrac{4}{3}\pi R^3 \rho g\right]$

Now considering the equilibrium of the heavy sphere

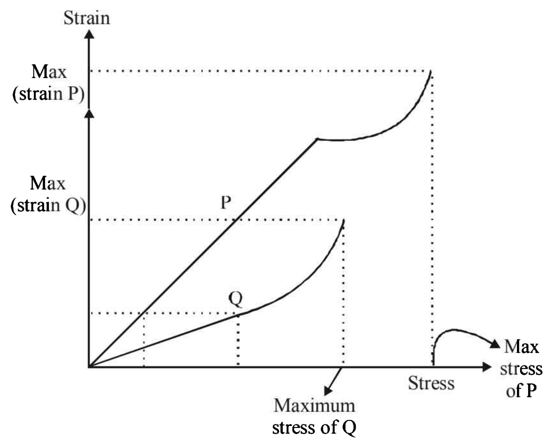
$Fs + B = W$

$\therefore Fs = W - B$

$\therefore Kx = \dfrac{4}{3}\pi R^3 (3\rho) g - \dfrac{4}{3}\pi R^3 (2\rho) g$

$\therefore x = \dfrac{4}{3}\pi \dfrac{R^3 \rho g}{K}$

7. **(a, b)** The maximum stress that P can withstand before breaking is greater than *Q*. Therefore (A) is a correct option.

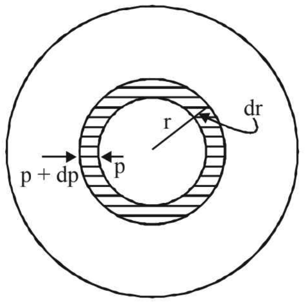


The strain of *P* is more than *Q* therefore *P* is more ductile. Therefore (B) is a correct option.

$Y = \dfrac{\text{stress}}{\text{strain}}$

For a given strain, stress is more for *Q*. Therefore $Y_Q > Y_P$.

8. **(b, c)** Let us consider an elemental mass dm shown in the shaded portion.



Here

$P\,4\pi r^2 - (P + dP)\,4\pi r^2$

$= \dfrac{GMr}{R^3}\,\rho\,(4\pi r^2)\,dr$

$\therefore\ -\int_0^P dp = \dfrac{GM\rho}{R^3}\int_R^r r\,dr$

$\therefore\ P = \dfrac{GM\rho}{2R^3}\left[R^2 - r^2\right]$

$\therefore\ \dfrac{P\,(r = 3R/4)}{P\,(r = 2R/3)} = \dfrac{\left[R^2 - \dfrac{9R^2}{16}\right]}{\left[R^2 - \dfrac{4R^2}{9}\right]} = \dfrac{\dfrac{7R^2}{16}}{\dfrac{5R^2}{9}} = \dfrac{63}{80}$

and $\dfrac{P\,(r = 3R/5)}{P\,(r = 2R/5)} = \dfrac{\left[R^2 - \dfrac{9R^2}{25}\right]}{\left[R^2 - \dfrac{4R^2}{25}\right]} = \dfrac{16}{21}$

B and *C* are correct options.

9. **(a, d)** From the figure it is clear that

(a) $\sigma_2 > \sigma_1$

(b) $\rho_2 > \sigma_2$ [As the string is taut]

(c) $\rho_1 < \sigma_1$ [As the string is taut]

$\therefore\ \rho_1 < \sigma_1 < \sigma_2 < \rho_2$

When P alone is in L_2

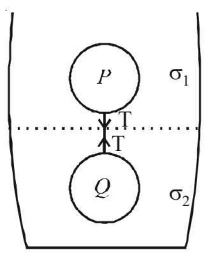
$V_P = \dfrac{2\pi r^2\,(\rho_1 - \sigma_2)\,g}{9\eta_2}$ is negative as $\rho_1 < \sigma_2$

Where r is radius of sphere.

When Q alone is in L_1

$V_Q = \dfrac{2\pi r^2\,(\rho_2 - \sigma_1)\,g}{9\eta_1}$ is positive as $\rho_2 > \sigma_1$

Therefore $\vec{V}_P.\vec{V}_Q < O$ option (d) is correct



Also $\dfrac{V_P}{V_Q} = \dfrac{\rho_1 - \sigma_2}{\rho_2 - \sigma_1} \times \dfrac{\eta_1}{\eta_2}$ ...(i)

For equilibrium of Q

$$T+\frac{4}{3}\pi r^3\ \sigma_2 g=\frac{4}{3}\pi r^3\ \rho_2 g \qquad ...(ii)$$

For equilibrium of P

$$T+\frac{4}{3}\pi r^3\ \rho_1 g=\frac{4}{3}\pi r^3\ \sigma_1 g \qquad ...(iii)$$

(iii) – (ii) gives

$$\rho_1-\sigma_2=\sigma_1-\rho_2 \qquad ...(iv)$$

From (i) and (iv)

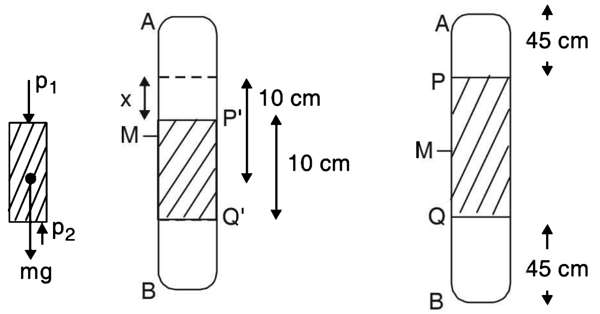
$$\frac{V_P}{V_Q}=-\frac{\eta_1}{\eta_2} \qquad \therefore \qquad \frac{|V_P|}{|V_Q|}=\frac{\eta_1}{\eta_2}$$

$\therefore$ A is also a correct option

E. Subjective Problems

1. M is the mid-point of tube AB.
At equilibrium

$$p_1\times A+mg=p_2\times A$$
$$p_1\times A+10\times A\times d_{Hg}g=p_2\times A$$
$$\Rightarrow\ p_1+10d_{Hg}\times g=p_2 \qquad ...(i)$$



For air present in column AP

$$p\times 45\times A=p_1\times(45+x)\times A$$
$$\Rightarrow\ p_1=\frac{45}{45+x}\times 76d_{Hg}\times g \qquad ...(ii)$$

For air present in column QB

$$p\times 45\times A=p_2\times(45-x)\times A$$
$$\Rightarrow\ p_2=\frac{45}{45-x}\times 76d_{Hg}\times g \qquad ...(iii)$$

From (i), (ii) and (iii)

$$\frac{45\times 76\times d_{Hg}g}{45+x}+10d_{Hg}\times g=\frac{45}{45-x}\times 76\times d_{Hg}\times g$$
$$\Rightarrow\ \frac{45\times 76}{45+x}+10=\frac{45\times 76}{45-x}$$

$x=2.95$ cm.

2. From fig. (b), due to equilibrium

$$T=mg \qquad ...(i)$$

But $Y=\dfrac{T/A}{\ell/L}$

$$\Rightarrow T=\frac{YA\ell}{L} \qquad ...(ii)$$

From (i) and (ii)

$$mg=\frac{YA\ell}{L} \qquad ...(iii)$$

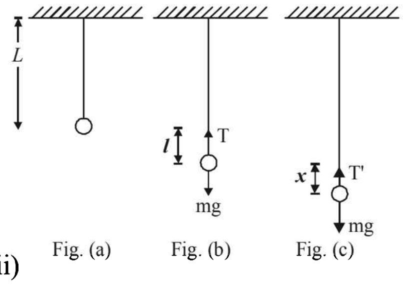


Fig. (a) Fig. (b) Fig. (c)

From fig. (c)
Restoring force

$$=-[T'-mg]=-\left[\frac{YA(\ell+x)}{L}-\frac{YA\ell}{L}\right] \quad \text{[from (iii)]}$$
$$=\frac{-YAx}{L}$$

On comparing this equation with $F=-m\omega^2x$, we get

$$m\omega^2=\frac{YA}{L}\ \Rightarrow \omega=\sqrt{\frac{YA}{mL}}\ \Rightarrow\ \frac{2\pi}{T}=\sqrt{\frac{YA}{mL}}$$

Frequency $f=\dfrac{1}{T}=\dfrac{1}{2\pi}\sqrt{\dfrac{YA}{mL}}$

3. Let the edge of cube be ℓ. When mass is on the cube of wood

$$200g+\ell^3 d_{wood}g=\ell^3 d_{water}g$$
$$\Rightarrow\ \ell^3 d_{wood}=\ell^3 d_{water}-200 \qquad ...(i)$$

When the mass is removed

$$\ell^3 d_{wood}=(\ell-2)\ell^2 d_{water} \qquad ...(ii)$$

From (i) and (ii)

$$\ell^3 d_{water}-200=(\ell-2)\ell^2 d_{water}$$

But $d_{water}=1$

$$\therefore\ \ \ell^3-200=\ell^2(\ell-2)\ \Rightarrow\ \ell=10\,\text{cm}$$

4. **KEY CONCEPT :**
When the stones were in the boat, the weight of stones were balanced by the buoyant force.

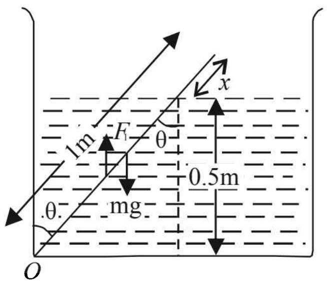
$$V_s d_s=V_\ell d_\ell$$

V_ℓ, V_s = volume of liquid and stone respectively
d_ℓ, d_s = density of liquid and stone respectively

Since, $d_s>d_\ell \quad \therefore\ V_s<V_\ell$

Therefore when stones are put in water, the level of water falls.

5. **(a)** For equilibrium $F_{net}=0$ and $\tau_{net}=0$



Taking moment about O

$$mg\times\frac{\ell}{2}\sin\theta=F_T\left(\frac{\ell-x}{2}\right)\sin\theta \qquad ...(i)$$

Also F_T = wt. of fluid displaced $=[(\ell-x)A]\times\rho_w g$... (ii)

And $m=(\ell A)0.5\rho_w$ ... (iii)

Where A is the area of cross section of the rod.
From (i), (ii) and (iii)

$$(\ell A)0.5\rho_w g\times\frac{\ell}{2}\sin\theta=[(\ell-x)A]\rho_w g\times\left(\frac{\ell-x}{2}\right)\sin\theta$$

Here, $\ell = 1$ m

$\therefore \quad (1-x)^2 = 0.5 \Rightarrow x = 0.293$ m

From the diagram

$$\cos\theta = \frac{0.5}{1-x} = \frac{0.5}{0.707} \Rightarrow \theta = 45°$$

6. **(a)** Let the ball be dropped from a height h. During fall

$$V = ut + at = 0 + g\frac{t_1}{2} \Rightarrow t_1 = \frac{2v}{g}$$

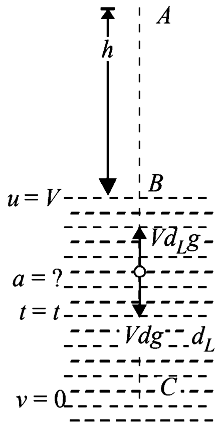
In the second case the ball is made to fall through the same height and then the ball strikes the surface of liquid of density d_L. When the ball reaches inside the liquid, it is under the influence of two force (i) Vdg, the weight of ball in downward direction (ii) Vd_Lg, the upthrust in upward direction.

Note :

The viscous forces are absent. (given)

Since, $d_L > d$

the upward force is greater and the ball starts retarding.



For motion B to C

$u = V,\ v = 0,\ t = t,\ a = -a$

$v = u + at \Rightarrow 0 = v + (-a)t$

$$\Rightarrow t = \frac{v}{a}$$

Now, $a = \dfrac{F_{net}}{m}$

$$= \frac{Vd_Lg - Vdg}{Vd} = \frac{(d_L - d)g}{d}$$

$$\Rightarrow t = \frac{vd}{(d_L - d)g} \quad \text{...(iii)}$$

Therefore,

$$t_2 = t_1 + 2t = t_1 + \frac{2dv}{(d_L - d)g}$$

$$= t_1 + \frac{2d}{(d_L - d)g}\frac{t_1 g}{2} = t_1\left[1 + \frac{d}{d_L - d}\right]$$

$$\Rightarrow t_2 = \frac{d_L t_1}{d_L - d}$$

(b) Since the retardation is not proportional to displacement, the motion of the ball is not simple harmonic.

(c) If $d = d_L$ then the retardation $a = 0$. Since the ball strikes the water surface with some velocity, it will continue with the same velocity in downward direction (until it is interrupted by some other force).

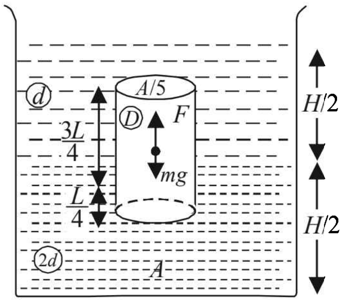
7. **(a)** (i)

KEY CONCEPT :

Since the cylinder is in equilibrium in the liquid therefore Weight of cylinder = upthrust

$mg = F_{T_1} + F_{T_2}$ where

F_{T_1} and F_{T_2} = upthrust due to lower and upper liquid respectively



$$\frac{A}{5} \times L \times D \times g = \frac{A}{5} \times \frac{L}{4} \times 2d \times g + \frac{A}{5} \times \frac{3L}{4} \times d \times g$$

$$\Rightarrow D = \frac{2d}{4} + \frac{3d}{4} = \frac{5d}{4}$$

(ii) Total pressure at the bottom of the cylinder = Atmospheric pressure + Pressure due to liquid of density d + Pressure due to liquid of density $2d$ + Pressure due to cylinder [Weight/Area]

$$P = P_0 + \frac{H}{2}dg + \frac{H}{2} \times 2d \times g + \frac{\frac{A}{5} \times L \times D \times g}{A}$$

$$\Rightarrow P = P_0 + \left(\frac{3H}{2} + \frac{L}{4}\right)dg \quad \left[\because D = \frac{5d}{4}\right]$$

(b) **KEY CONCEPT :**

Applying Bernoulli's theorem

$$P_0 + \left[\frac{H}{2} \times d \times g + \left(\frac{H}{2} - h\right)2d \times g\right]$$

$$= P_0 + \frac{1}{2}(2d)v^2 \Rightarrow v = \sqrt{\frac{(3H - 4h)}{2}g}$$

Horizontal Distance x

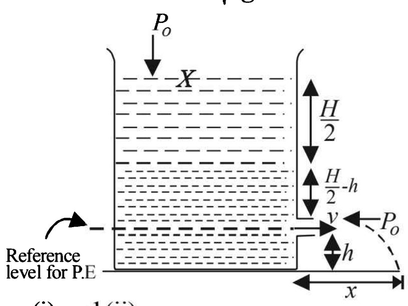
$u_x = v;\ t = t;\quad x = vt$ .... (i)

For vertical motion of liquid falling from hole

$u_y = 0, S_y = h, a_y = g, t_y = t$

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} \quad \text{...(ii)}$$



From (i) and (ii)

$$x = v_y \times \sqrt{\frac{2h}{g}} = \sqrt{(3H - 4h)\frac{g}{2}} \times \sqrt{\frac{2h}{g}}$$

$$= \sqrt{(3H - 4h)h} \quad \text{... (iii)}$$

For finding the value of h for which x is maximum, we differentiate equation (iii) w.r.t. t

$$\frac{dx}{dt} = \frac{1}{2}[3H - 4h)h]^{-1/2}\{3H - 8h\}$$

Putting $\frac{dx}{dt} = 0$ for finding values h for maxima/minima

$$\frac{1}{2}[(3H-4h)]^{-1/2}[3H-8h] = 0 \Rightarrow h = \frac{3H}{8}$$

$$\therefore \quad x_m = \sqrt{\left[3H - 4\left(\frac{3H}{8}\right)\right]\frac{3H}{8}} \qquad \text{[From (iii)]}$$

$$= \sqrt{\frac{12H}{8} \times \frac{3H}{8}} = \frac{6H}{8} = \frac{3H}{4}$$

8. Given that

$\rho = 1000\ \text{kg/m}^3, h_1 = 2\text{m}, h_2 = 5\ \text{m}$

$A_1 = 4 \times 10^{-3}\text{m}^2, A_2 = 8 \times 10^{-3}\ \text{m}^2, v_1 = 1\ \text{m/s}$

Equation of continuity

$$A_1 v_1 = A_2 v_2 \quad \therefore v_2 = \frac{A_1 v_1}{A_2} = 0.5\ \text{m/s}$$

According to Bernoulli's theorem,

$$(p_1 - p_2) = \rho g\,(h_2 - h_1) - \frac{1}{2}\rho(v_2^2 - v_1^2)$$

Where $(p_1 - p_2)$ = work done/vol. [by the pressure]

$\rho g\,(h_2 - h_1)$ = work done/vol. [by gravity forces]

Now, work done/vol. by gravity forces

$= \rho g\,(h_2 - h_1) = 10^3 \times 9.8 \times 3 = 29.4 \times 10^3\ \text{J/m}^3.$

And $\frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2} \times 10^3\left[\frac{1}{4} - 1\right] = -\frac{3}{8} \times 10^3\ \text{J/m}^3$

$= -0.375 \times 10^3\ \text{J/m}^3$

$\therefore$ Work done / vol. by pressure

$= 29.4 \times 10^3 - 0.375 \times 10^3\ \text{J/m}^3 = 29.025 \times 10^3\ \text{J/m}^3.$

9. **(a)** As the pressure exerted by liquid A on the cylinder is radial and symmetric, the force due to this pressure cancels out and the net value is zero.

(b) For equilibrium, Buoyant force = weight of the body

$\Rightarrow h_A \rho_A A g + h_B \rho_B A g = (h_A + h + h_B)\, A\, \rho_C g$

(where ρ_C = density of cylinder)

$$h = \left(\frac{h_A\rho_A + h_B\rho_B}{\rho_C}\right) - (h_A + h_B) = 0.25\ \text{cm}$$

(c) $a = \frac{F_{\text{Buoyant}} - Mg}{M}$

$$= \left[\frac{h_A\rho_A + \rho_B(h + h_B) - (h + h_A + h_B)\rho_C}{\rho_C(h + h_A + h_C)}\right] g$$

$= \frac{g}{6}$ upwards

10. **KEY CONCEPT :** When the force due to excess pressure in the bubble equals the force of air striking at the bubble, the bubble will detach from the ring.

$$\therefore \quad \rho A v^2 = \frac{4T}{R} \times A \quad \Rightarrow R = \frac{4T}{\rho v^2}$$

11. **KEY CONCEPT :** When the tube is not there, using Bernoulli's theorem

$$P + P_0 + \frac{1}{2}\rho v_1^2 + \rho g H = \frac{1}{2}\rho v_0^2 + P_0$$

$$\Rightarrow \quad P + \rho g H = \frac{1}{2}\rho(v_0^2 - v_1^2)$$

But according to equation of continuity

$$v_1 = \frac{A_2 v_0}{A_1}$$

$$\therefore \quad P + \rho g H = \frac{1}{2}\rho\left[v_0^2 - \left(\frac{A_2}{A_1}v_0\right)^2\right]$$

$$P + \rho g H = \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

Here, $P + \rho g H = \Delta P$

According to Poisseuille's equation

$$Q = \frac{\pi(\Delta P)a^4}{8\eta l} \Rightarrow \eta = \frac{\pi(\Delta P)a^4}{8Q\ell}$$

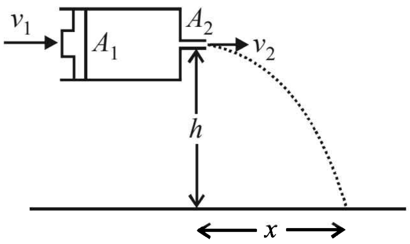
$$\therefore \quad \eta = \frac{\pi(P + \rho g H)a^4}{8Q\ell} = \frac{\pi}{8Q\ell} \times \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] \times a^4$$

Where $\frac{A_2}{A_1} = \frac{d^2}{D^2}$

$$\eta = \frac{\pi}{8Ql} \times \frac{1}{2}\rho v_0^2\left[1 - \frac{d^4}{D^4}\right] \times a^4$$

12. From law of continuity $A_1 v_1 = A_2 v_2$

Given $A_1 = \pi \times (4 \times 10^{-3}\ \text{m})^2,\ A_2 = \pi \times (1 \times 10^{-3}\ \text{m})^2$



$v_1 = 0.25\ \text{m/s}$

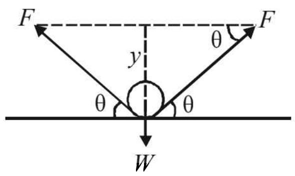
$$\therefore \quad v_2 = \frac{\pi \times (4 \times 10^{-3})^2 \times 0.25}{\pi \times (1 \times 10^{-3})^2} = 4\ \text{m/s}$$

Also, $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

Horizontal range $x = v_2 \times t = v_2\sqrt{\frac{2h}{g}} = 4 \times \sqrt{\frac{2 \times 1.25}{10}} = 2\text{m}$

13. The free body diagram of wire is given below. If ℓ is the length of wire, then for equilibrium $2F\sin\theta = W$.

$F = S \times \ell$

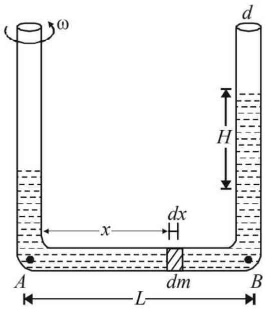


or, $2S \times \ell \times \sin\theta = \lambda \times \ell \times g$

or, $S = \frac{\lambda g}{2\sin\theta}$

$$\therefore \quad S = \frac{\lambda g}{2y/a} = \frac{a\lambda g}{2y} \qquad \left[\because \sin\theta = \frac{y}{a}\right]$$

14. Weight of liquid of height H



$$= \frac{\pi d^2}{4} \times H \times \rho \times g \text{ ... (i)}$$

Let us consider a mass dm situated at a distance x from A as shown in the figure. The centripetal force required for the mass to rotate $= (dm)\, x\omega^2$

$\therefore$ The total centripetal force required for the mass of length L to rotate

$$= \int_0^L (dm)\, x\omega^2 \quad \text{where } dm = \rho \times \frac{\pi d^2}{4} \times dx$$

$$\therefore \text{ Total centripetal force} = \int_0^L \left(\rho \times \frac{\pi d^2}{4} \times dx\right) \times \left(x\omega^2\right)$$

$$= \rho \times \frac{\pi d^2}{4} \times \omega^2 \int_0^L x\, dx$$

$$= \rho \times \frac{\pi d^2}{4} \times \omega^2 \times \frac{L^2}{2} \qquad \text{... (ii)}$$

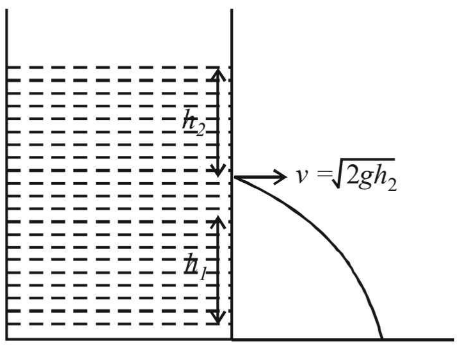
This centripetal force is provided by the weight of liquid of height H.

From (i) and (ii)

$$\frac{\pi d^2}{4} \times H \times \rho \times g = \rho \times \frac{\pi d^2}{4} \times \frac{\omega^2 \times L^2}{2} \Rightarrow H = \frac{\omega^2 L^2}{2g}$$

F. Match the Following

1. **(c)**



$$\sqrt{2gh_2} \times \sqrt{\frac{2h_1}{g}}$$

$$= 2\sqrt{h_1 h_2}$$

If $g_{eff} > g$

$g_{eff} = g$

$g_{eff} < g$

In all the three cases $d = 2\sqrt{h_1 h_2} = 1.2$ m

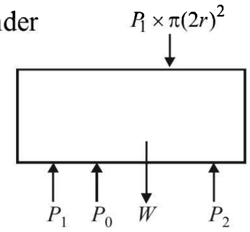
If $g_{eff} = 0$, then no water leaks out

G. Comprehension Based Questions

1. **(c)** Consider the equilibrium of wooden block.

Forces acting in the downward direction are

(a) Weight of wooden cylinder



$$= \pi(2r)^2 \times h \times \frac{\rho}{3} \times g$$

$$= \pi \times 4r^2 \frac{h\rho}{3} g$$

(b) Force due to pressure (P_1) created by liquid of height h_1 above the wooden block is

$= P_1 \times \pi(2r)^2 = [P_0 + h_1\rho g] \times \pi(2r)^2$

Force acting on the upward direction due to pressure P_2 exerted from below the wooden block and atmospheric pressure is

$$= P_2 \times \pi\left[(2r)^2 - r^2\right] + P_0 \times \pi(r)^2$$

$$= \left[P_0 + (h_1 + h)\rho g\right] \times \pi \times 3r^2 + P_0 \pi r^2$$

At the verge of rising

$$\left[P_0 + (h_1 + h)\rho g\right] \times \left(\pi \times 3r^2\right) + \pi r^2 P_0$$

$$= \left[P_0 + h_1\rho g\right] \times 4\pi r^2 + \frac{\pi \times 4r^2 h\rho g}{3} \quad \text{or,} \quad h_1 = \frac{5h}{3}$$

2. **(b)** **KEY CONCEPT :**

Considering equilibrium of wooden block.

Total downward force = Total force upwards

Wt. of block + force due to atmospheric pressure = Force due to pressure of liquid + Force due to atmospheric pressure

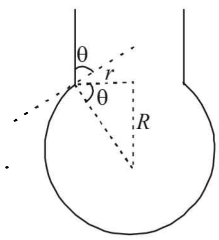
$$\pi(16r^2)\frac{\rho}{3} \times g + P_0 \pi \times 16r^2$$

$$= [h_2\rho g + P_0]\, \pi\, [(16-4)r^2] + P_0 \times 4r^2$$

$$\Rightarrow \frac{4}{9}h = h_2$$

3. **(a)** When the height h_2 of water level is further decreased, then the upward force acting on the wooden block decreases. The total force downward remains the same. This difference will be compensated by the normal reaction by the tank wall on the wooden block. Thus the block does not moves up and remains at its original position.

4. **(c)** The vertical force due to surface tension



$= (T\cos\theta) \times 2\pi r$

$$= T\left(\frac{r}{R}\right) \times 2\pi r = \frac{2\pi r^2 T}{R}$$

5. **(a)** When the drop is about to detach from the dropper

Weight = vertical force due to surface tension

$$\frac{4}{3}\pi R^3 \rho g = \frac{2\pi r^2 T}{R}$$

$$\therefore R^4 = \left(\frac{3}{2}\frac{r^2 T}{\rho g}\right) = \frac{3}{2} \times \frac{(5 \times 10^{-4})^2 \times 0.11}{1000 \times 10} = 4.12 \times 10^{-12}$$

$\therefore R = 1.42 \times 10^{-3}$ m

6. (b) We know that, $T = \frac{U}{a}$

$\therefore U = T \times a = T \times 4\pi R^2$

$= 0.11 \times 4 \times \frac{22}{7} \times (1.42 \times 10^{-3})^2 = 2.7 \times 10^{-6} J$

7. (c) From principle of continuity,

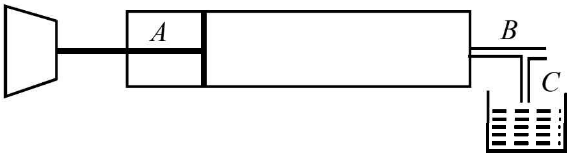
$a_1 v_1 = a_2 v_2$

$\pi r_1^2 v_1 = \pi r_2^2 v_2$

$(20)^2 \times 5 = (1)^2 \times v_2$

$\therefore v_2 = 2000 \text{ mms}^{-1} = 2 \text{ ms}^{-1}$

8. (a)



$$P_A - P_B = \frac{1}{2}\rho_a v_a^2$$

$$P_C - P_B = \frac{1}{2}\rho_l v_l^2$$

But $P_C = P_A$

$$\therefore \frac{1}{2}\rho_l v_l^2 = \frac{1}{2}\rho_a v_a^2 \Rightarrow v_l = \sqrt{\frac{\rho_a}{\rho_l}} \times v_a$$

$\therefore$ Volume flow rate $\propto \sqrt{\frac{\rho_a}{\rho_l}}$

H. Assertion & Reason Type Questions

1. (a) We know that volume flow rate (V) of an incompressible fluid in steady flow remains constant.

$$V = a \times v$$

where a = area of cross-section and v = velocity

$\Rightarrow$ If v decreases a increases and vice - versa.

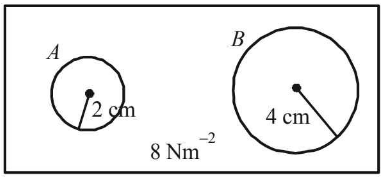
When stream of water moves up, its speed (v) decreases and therefore 'a' increases i.e. the water spreads out as a fountain. When stream of water from hose pipe moves down, its speed increases and therefore area of cross-section decreases.

Therefore statement-1 is true and statement-2 is the correct explanation of statement-1.

I. Integer Value Correct Type

1. **For bubble A:**

If P_A is the pressure inside the bubble then



$$P_A - 8 = \frac{4T}{R_A} = \frac{4 \times 0.04}{0.02} = 8 \Rightarrow P_A = 16 N/m^2$$

According to ideal gas equation,

$$P_A V_A = n_A R T_A \Rightarrow 16 \times \frac{4}{3}\pi(0.02)^3 = n_A R T_A \quad \ldots (i)$$

For bubble B:

If P_B is the pressure inside the bubble then

$$P_B - 8 = \frac{4T}{R_B} = \frac{4 \times 0.04}{0.04} = 4 \Rightarrow P_B = 12 N/m^2$$

According to ideal gas equation

$$P_B V_B = n_B R T_B \Rightarrow 12 \times \frac{4}{3}\pi(0.04)^3 = n_B R T_B \ldots (ii)$$

Dividing (ii) by (i) we get

$$\frac{12 \times \frac{4}{3}\pi(0.04)^3}{16 \times \frac{4}{3}\pi(0.02)^3} = \frac{n_B}{n_A} \quad [\because T_A = T_B]$$

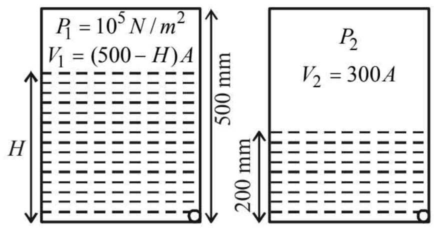
$$\therefore \frac{n_B}{n_A} = 6$$

2. Initially, the pressure of air column above water is $P_1 = 10^5$ Nm^{-2} and volume $V_1 = (500 - H)A$, where A is the area of cross-section of the vessel.

Finally, the volume of air column above water is 300 A. If P_2 is the pressure of air then

$$P_2 + \rho g h = 10^5$$

$$\therefore P_2 + 10^3 \times 10 \times \frac{200}{1000} = 10^5$$



Initially Finally

$$\therefore P_2 = 9.8 \times 10^4 N/m^2$$

As the temperature remains constant, according to Boyle's law

$$P_1 V_1 = P_2 V_2$$

$\therefore 10^5 \times (500 - H)A = (9.8 \times 10^4) \times 300A \Rightarrow H = 206$ mm

$\therefore$ The fall of height of water level due to the opening of orifice $= 206 - 200 = 6$ mm

3. We know that $\omega = \sqrt{\frac{K}{m}}$ ...(i)

Here $Y = \frac{FL}{Al} \Rightarrow F = \left(\frac{YA}{L}\right) l$

Comparing the above equation with F = kl we get

$K = \left(\frac{YA}{L}\right)$ ...(ii)

From (i) & (ii), $\omega = \sqrt{\frac{YA}{ml}}$

$\therefore 140 = \sqrt{\frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}} \quad \therefore n = 4$

4. 3 $$\frac{V_P}{V_Q} = \frac{\frac{2r_1^2(\sigma - \rho_1)g}{9\eta_1}}{\frac{2r_2^2(\sigma - \rho_2)g}{9\eta_2}} = \frac{r_1^2(\sigma - \rho_1)}{r_2^2(\sigma - \rho_2)} \times \frac{\eta_2}{\eta_1}$$

$$= \frac{1^2}{(0.5)^2} \frac{[8 - 0.8]}{[8 - 1.6]} \times \frac{2}{3} = 3$$

Section-B JEE Main/ AIEEE

1. **(b)** Small amount of work done in extending the spring by dx is

$dW = k\,x\,dx$

$$\therefore W = k\int_{0.05}^{0.15} x\,dx = \frac{800}{2}\left[(0.15)^2 - (0.05)^2\right] = 8\text{ J}$$

2. **(d)** Work done by constant force in displacing the object by a distance ℓ.

$$= \frac{1}{2}\times \text{Force}\times\text{extension} = \frac{F\ell}{2}$$

3. **(b)** From Stoke's law,

viscous force $F = 6\pi\eta r v$

hence F is directly proportional to radius & velocity.

4. **(a)** $P_0 - P_i = \dfrac{2T}{R}$

$\therefore P_1 > P_2$

hence air moves from smaller bubble to bigger bubble.

5. **(a)** Energy stored per unit volume,

$$E = \frac{1}{2}\times\text{stress}\times\text{strain}$$

$$\therefore E = \frac{1}{2}\times\text{stress}\times\frac{\text{stress}}{Y} = \frac{1}{2}\cdot\frac{S^2}{Y}$$

6. **(c)** Water fills the tube entirely in gravity less condition i.e., 20 cm.

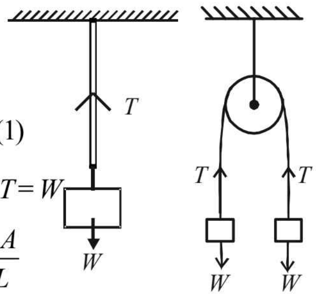
7. **(a)** Case (i)

At equilibrium, $T = W$

$$Y = \frac{W/A}{\ell/L} \quad .....(1)$$

Case (ii) At equilibrium $T = W$

$$\therefore Y = \frac{W/A}{\frac{\ell/2}{L/2}} \Rightarrow Y = \frac{W/A}{\ell/L}$$



$\Rightarrow$ Elongation is the same.

8. **(c)** Terminal velocity, $v_T = \dfrac{2r^2(d_1-d_2)g}{9\eta}$

$$\frac{v_{T_2}}{0.2} = \frac{(10.5-1.5)}{(19.5-1.5)} \Rightarrow v_{T_2} = 0.2\times\frac{9}{18} = 0.1\text{ m/s}$$

9. **(a)** The condition for terminal speed (v_t) is

Weight = Buoyant force + Viscous force

$$\therefore V\rho_1 g = V\rho_2 g + kv_t^2 \qquad \therefore v_t = \sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$$

10. **(d)** From the figure it is clear that liquid 1 floats on liquid 2. The lighter liquid floats over heavier liquid. Therefore we can conclude that $\rho_1 < \rho_2$

Also $\rho_3 < \rho_2$ otherwise the ball would have sink to the bottom of the jar.

Also $\rho_3 > \rho_1$ otherwise the ball would have floated in liquid 1. From the above discussion we conclude that

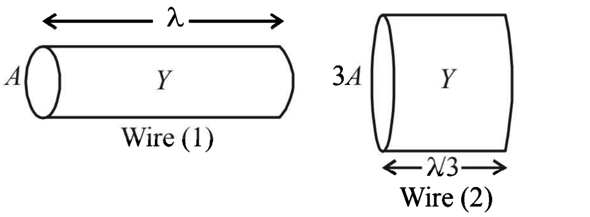
$$\rho_1 < \rho_3 < \rho_2.$$

11. **(c)** In case of water, the meniscus shape is concave upwards. Also according to ascent formula

$$h = \frac{2T\cos\theta}{r\rho g}$$

The surface tension (T) of soap solution is less than water. Therefore rise of soap solution in the capillary tube is less as compared to water. As in the case of water, the meniscus shape of soap solution is also concave upwards.

12. **(c)**



As shown in the figure, the wires will have the same Young's modulus (same material) and the length of the wire of area of cross-section $3A$ will be $\ell/3$ (same volume as wire 1).

For wire 1,

$$Y = \frac{F/A}{\Delta x/\ell} \quad ...(i)$$

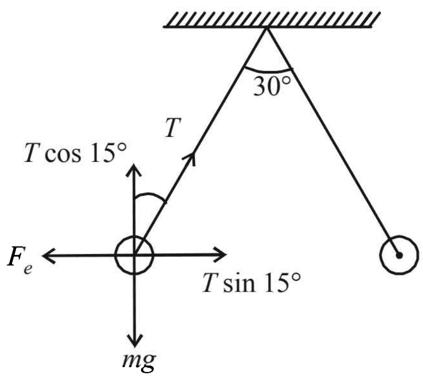
For wire 2,

$$Y = \frac{F'/3A}{\Delta x/(\ell/3)} \quad ...(ii)$$

From (i) and (ii), $\dfrac{F}{A}\times\dfrac{\ell}{\Delta x} = \dfrac{F'}{3A}\times\dfrac{\ell}{3\Delta x} \Rightarrow F' = 9F$

13. **(b)** Oil will float on water so, (2) or (4) is the correct option. But density of ball is more than that of oil,, hence it will sink in oil.

14. **(c)**

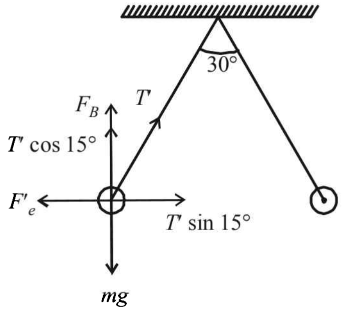


$$F_e = T\sin 15° \quad ; \quad mg = T\cos 15°$$

$\Rightarrow \tan 15° = \dfrac{F_e}{mg}$ ...(1)

In liquid, $F_e' = T'\sin 15°$ ...(A)

$mg = F_B + T'\cos 15°$



$F_B = V(d-\rho)g = V(1.6-0.8)g = 0.8\,Vg$

$= 0.8\dfrac{m}{d}g = \dfrac{0.8mg}{1.6} = \dfrac{mg}{2}$

$\therefore mg = \dfrac{mg}{2} + T'\cos 15°$

$\Rightarrow \dfrac{mg}{2} = T'\cos 15°$ ...(B)

From (A) and (B), $\tan 15° = \dfrac{2F_e'}{mg}$ ... (2)

From (1) and (2)

$\dfrac{F_e}{mg} = \dfrac{2F_e'}{mg} \Rightarrow F_e = 2F_e' \Rightarrow F_c' = \dfrac{F_c}{2}$

15. (c) $W = T \times$ change in surface area

$W = 2T4\pi[(5^2)-(3)^2]\times 10^{-4}$

$= 2\times 0.03\times 4\pi[25-9]\times 10^{-4}\text{ J} = 0.4\pi\times 10^{-3}\text{ J}$

$= 0.4\pi$ mJ

16. (c) From Bernoulli's theorem,

$P_0 + \frac{1}{2}\rho v_1^2 \rho gh = P_0 + \frac{1}{2}\rho v_2^2 + 0$

$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{0.16 + 2\times 10\times 0.2} = 2.03$ m/s

From equation of continuity

$A_2v_2 = A_1v_1$

$\pi\dfrac{D_2^2}{4}\times v_2 = \pi\dfrac{D_1^2}{4}v_1$

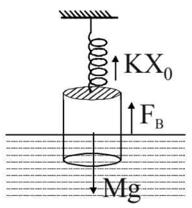
$\Rightarrow D_1 = D_2\sqrt{\dfrac{v_1}{v_2}} = 3.55\times 10^{-3}$ m

17. (d) At equilibrium,

$2Tl = mg$

$T = \dfrac{mg}{2l} = \dfrac{1.5\times 10^{-2}}{2\times 30\times 10^{-2}} = \dfrac{1.5}{60} = 0.025$ N/m $= 0.025$Nm

18. (c) From figure, $kx_0 + F_B = Mg$



$kx_0 + \sigma\dfrac{L}{2}Ag = Mg$

[$\because$ mass = density × volume]

$\Rightarrow kx_0 = Mg - \sigma\dfrac{L}{2}Ag$

$\Rightarrow x_0 = \dfrac{Mg - \dfrac{\sigma LAg}{2}}{k} = \dfrac{Mg}{k}\left(1 - \dfrac{LA\sigma}{2M}\right)$

19. (d) When radius is decrease by ΔR,

$4\pi R^2\Delta R\rho L = 4\pi T[R^2 - (R-\Delta R)^2]$

$\Rightarrow \rho R^2\Delta RL = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$

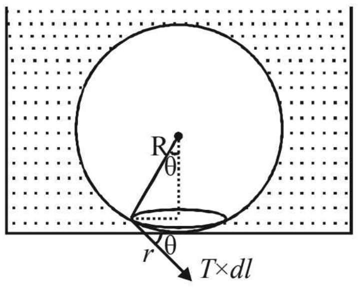
$\Rightarrow \rho R^2\Delta RL = T2R\Delta R$ [ΔR is very small]

$\Rightarrow R = \dfrac{2T}{\rho L}$

20. (None) None of the given option is correct.

When the bubble gets detached,

Buoyant force = force due to surface tension



Force due to excess pressure = upthrust

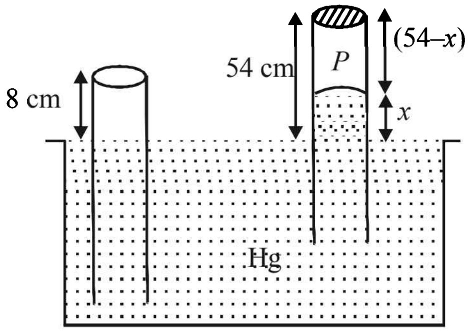
Access pressure in air bubble = $\dfrac{2T}{R}$

$\dfrac{2T}{R}(\pi r^2) = \dfrac{4\pi R^3}{3T}\rho_w g$

$\Rightarrow r^2 = \dfrac{2R^4\rho_w g}{3T}$

$\Rightarrow r = R^2\sqrt{\dfrac{2\rho_w g}{3T}}$

21. (a)



Length of the air column above mercury in the tube is,

$P + x = P_0$

$\Rightarrow P = (76-x)$

$\Rightarrow 8\times A\times 76 = (76-x)\times A\times(54-x)$

$\therefore x = 38$

Thus, length of air column = 54 − 38 = 16 cm.